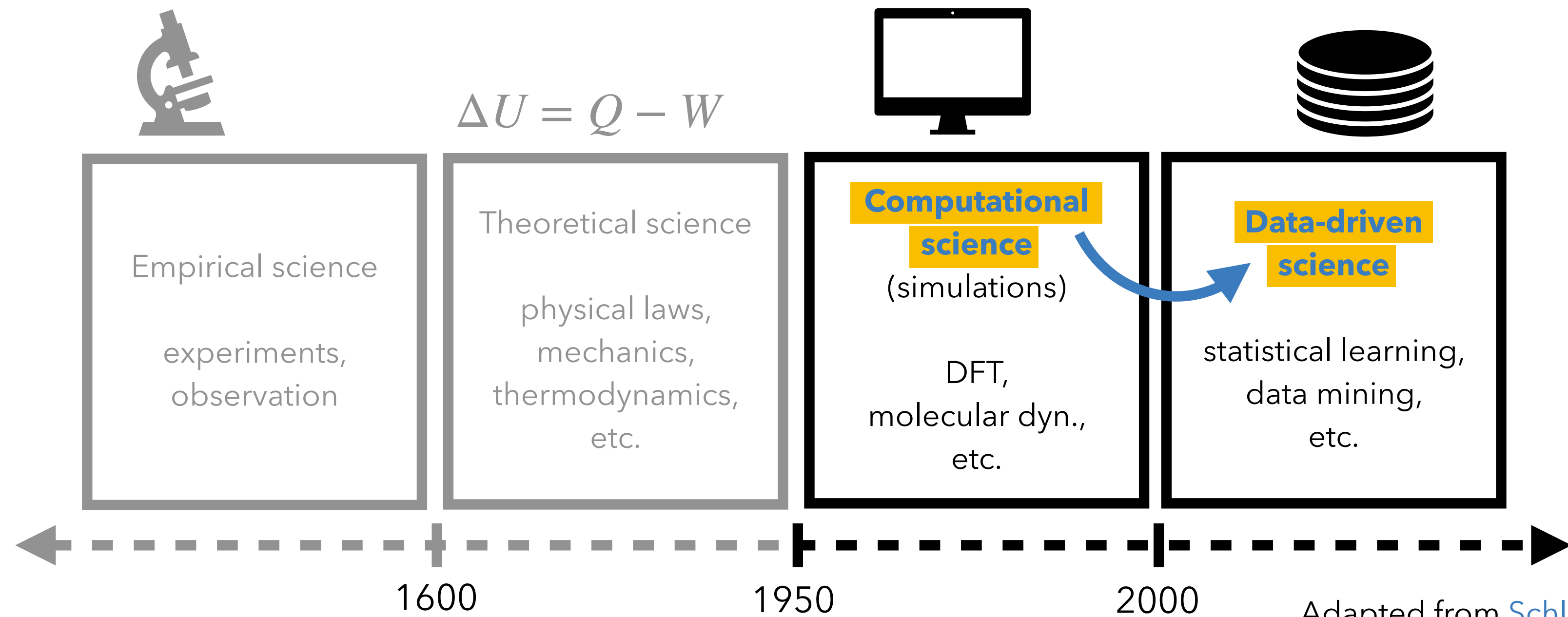


A Benchmark of
predicting magnetic structures using
a Combination of the
Cluster Multipole Expansion and LSDA

M.-T. Huebsch , T. Nomoto , M.-T. Suzuki and R. Arita



New Paradigm: Data-Driven Science



Data-driven material design:

Materials Project, AFLOW, ICSD, Clean Energy Project, ...

Ex. of application:

carbon capturing [[Dunstan et al., Energy Environ. Sci. \(2016\)](#)],
battery efficiency [[Qu et al., Com. Mat. Sci. \(2015\)](#)],
thermoelectric materials [[Toher et al., PR Mat. \(2017\)](#)], ...

Potential in Non-Collinear Antiferromagnetism

Symmetry-imposed shape of linear response tensors [[Seemann et al., PRB \(2015\)](#), [Kleiner et al., PR \(1966\)](#), [PR \(1967\)](#)]

⇒ Anomalous response in AFM is possible,

e.g. in Mn_3Sn [[Nakatsuji et al., nat. \(2015\)](#)], Mn_3Ge [[Nayak et al., Sci. Adv. \(2016\)](#)],
 $\text{Mn}_3\text{Ni}_{1-x}\text{Cu}_x\text{N}$ [[Zhao et al., PRB \(2019\)](#)], ...

Advantages of non-collinear AFM:

- no stray field
- fast dynamics [[Nomoto and Arita, PR Research \(2019\)](#)]
- robustness against perturbations

Applications of non-collinear AFM:

- seamless and low-maintenance energy generation [[Ikhlas et al., nat. phys. \(2017\)](#)]
- ultrafast spintronics and robust data retention [[Baltz et al., Rev. Mod. Phys. \(2018\)](#)]
- fundamental understanding of magneto-transport

Computational Effort : Non-Collinear Antiferromagnetism



Local Spin-Density Approximation (LSDA) for non-collinear magnetism [[Barth and Hedin, J. Phys. C \(1972\)](#)]

Challenges :

- 3 continuous degrees of freedom
- many local minima in the LSDA free energy landscape
- LSDA global minimum = true ground state ?
- magnetic unit cell = crystallographic unit cell

Goal : Predict the magnetic ground state given a compound.

Benchmark needed :

- candidate magnetic configurations ← Cluster Multipole Expansion [[Suzuki et al., PRB \(2019\)](#)]
- reference data ← [MAGNDATA](#) [[Gallego et al., J. Appl. Cryst. \(2016\)](#)]
- high-throughput LSDA ← non-coll. VASP [[Hobbs and Hafner, J. Phys. Condes. Matter \(2001\)](#)],
[pymatgen](#) [[Ong et al., J. Com. Mat. Sci. \(2013\)](#)],
tools on the [Bilbao Crystallographic Server](#) [[Aroyo et al., Bul. Chem. Com. \(2011\)](#)]

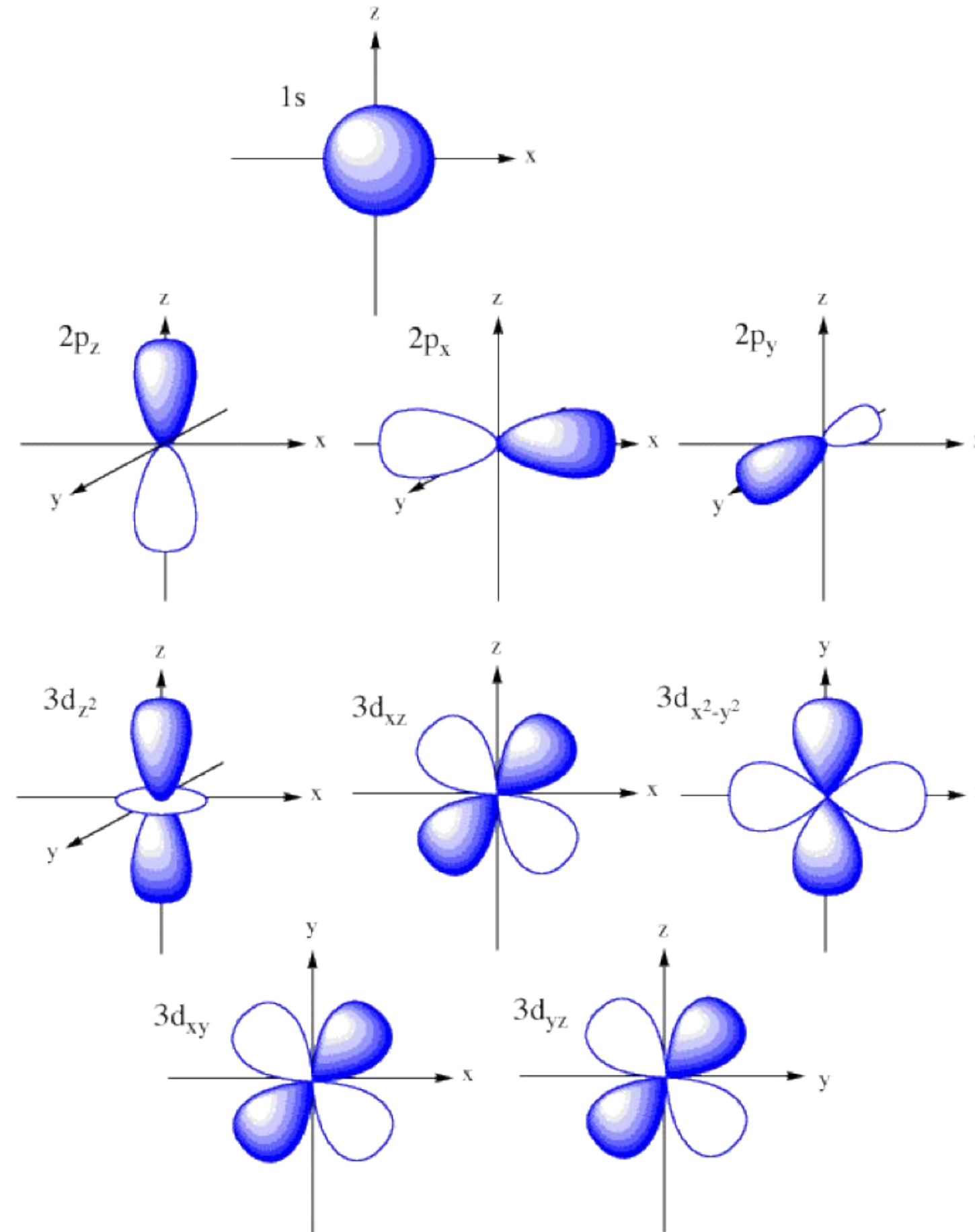
Multipole Expansion for an Electron Cloud

Scalar Poisson Equation

$$\nabla^2 \Phi(\mathbf{r}) = -4\pi\rho(\mathbf{r})$$

Scalar potential :

$$\Phi(\mathbf{r}) = \sum_{lm} \sqrt{\frac{4\pi}{2l+1}} Q_{lm} \frac{Y_{lm}}{r^{l+1}}$$



Vector Poisson Equation

$$\nabla^2 \mathbf{A}(\mathbf{r}) = - (4\pi/c) \mathbf{j}(\mathbf{r})$$

$$\mathbf{j}(\mathbf{r}) = c \nabla \times \mathbf{M}(\mathbf{r})$$

Vector potential :

$$\mathbf{A}(\mathbf{r}) = \sum_{lm} M_{lm} \frac{\mathbf{l} Y_{lm}}{i l} \frac{1}{r^{l+1}}$$

Angular momentum :

$$\mathbf{l} = -i(\mathbf{r} \times \nabla) \quad (l \geq 1, -l \leq m \leq l)$$

Multipole for an electron cloud :

$$M_{lm} = \sqrt{\frac{4\pi}{2l+1}} \sum_j \left(\frac{2\mathbf{l}_j}{l+1} + \sigma_j \right) \nabla \left[|\mathbf{r}_j|^l Y_{lm}^*(\varphi, \theta) \right]$$

Cluster Multipole (CMP) Expansion for Magnetic Structures

Multipole for an electron cloud:

$$M_{lm} = \sqrt{\frac{4\pi}{2l+1}} \sum_j \left(\frac{2l_j}{l+1} + \sigma_j \right) \cdot \nabla \left[|\mathbf{r}_j|^l Y_{lm}^*(\varphi, \theta) \right]$$

Multipole for a point form:

$$M_{lm} = \sqrt{\frac{4\pi}{2l+1}} \sum_{i=1}^N \mathbf{m}_i \cdot \nabla \left[|\mathbf{r}_i|^l Y_{lm}^*(\varphi, \theta) \right]$$

[Suzuki et al., PRB \(2019\)](#)

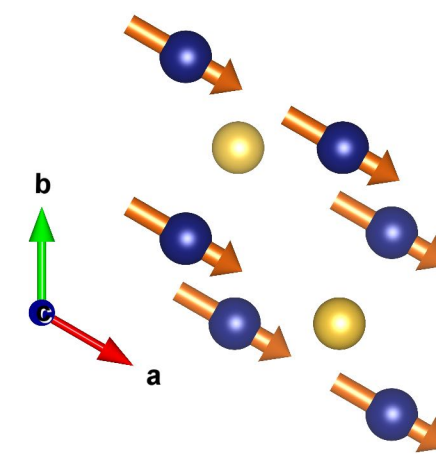
For a point group $\mathcal{P}$ a **point form** is a set of all symmetrically equivalent points.

N is the **multiplicity** of the point form.

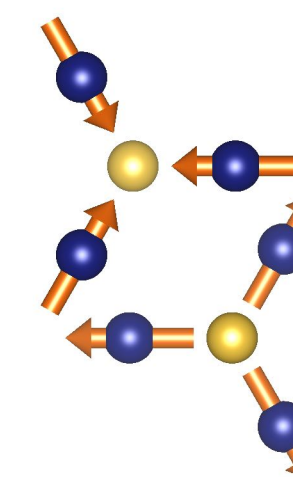
Point form + magnetic configuration = **(magnetic) cluster**.

Example of CMP configurations for Mn_3Sn :

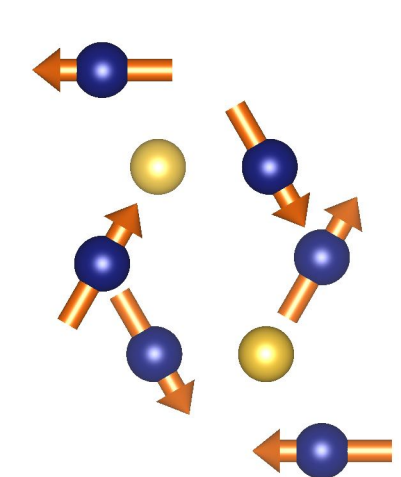
dipole



octupole

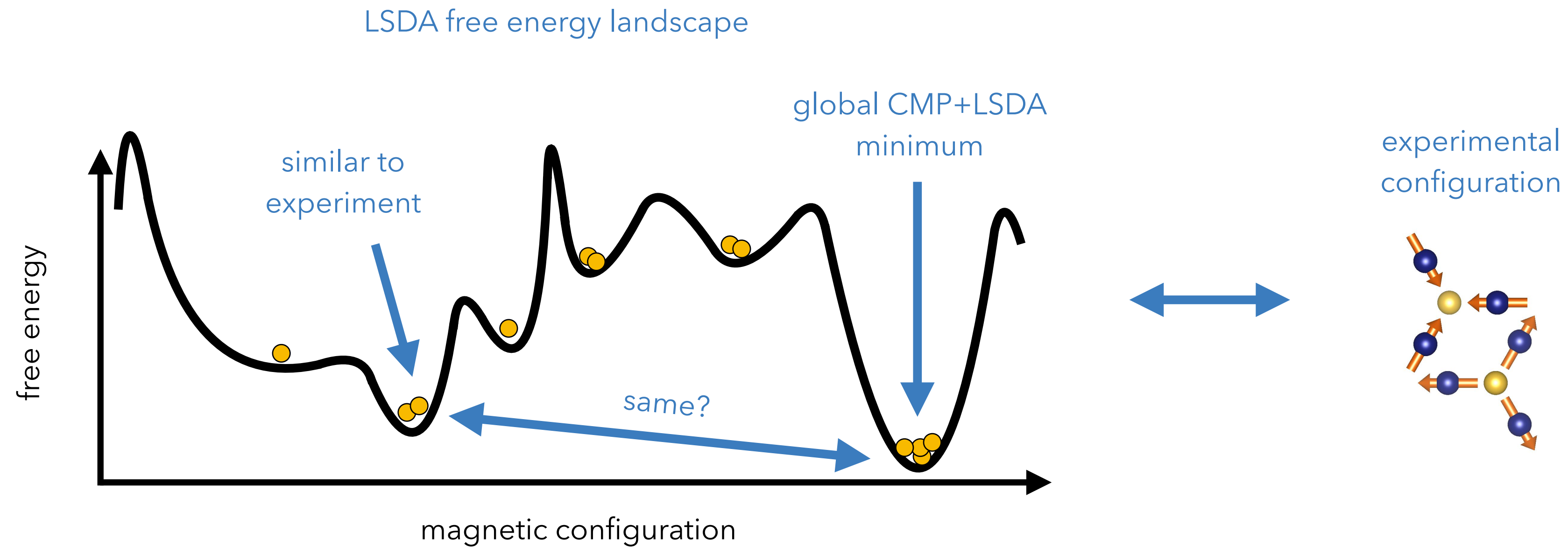


octupole



Each CMP carries a definite **order & irrep**.

High-Throughput CMP+LSDA Calculations



- Most magnetic configurations can be characterized by one or few symmetrically related CMPs.
- Experimental configuration is among the CMP+LSDA local minima.
- In about 1/3 of the cases: CMP+LSDA global minimum = experimental configuration.

Is the Magnetic Moment per Site well-estimated?

- No general over-/under-estimation
- No reliable prediction $\lesssim 2\mu_B$
- $\gtrsim 2\mu_B$ accurate prediction, with low precision

