

# Towards a Feature Space for Magnetic Structures

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Many calculations in solid state physics become dramatically more efficient by using an orthonormal basis adapted to the symmetry of the material. The **cluster multipole (CMP) expansion** for magnetic structures [1] provides an **orthonormal basis set of magnetic configurations based on the crystallographic point group**. Recent discussions in terms of CMPs established a novel perspective on transverse response properties [2] and spin dynamics in non-collinear antiferromagnets [3]. Our analysis of the experimental data found on MAGNDATA provided by Bilbao Crystallographic Server [4] suggests that only a few terms are finite in the CMP expansion of the most stable configurations in nature. This sparseness can be exploited to **build a larger learning database**, however that is provided that a **reliable prediction of the most stable configuration can be made—possibly by means of ab-initio calculations**.

## Build a database of magnetic structures,

- ▶ obeying the 5 V's of Big Data [5]
- ▶ using CMP expansion to create candidate magnetic configurations [1]
- ▶ determining the most stable configuration by LSDA [6] using VASP [7]

## Antiferromagnetic materials are particularly interesting [8], because they

- ▶ are robust against perturbation due to magnetic fields,
- ▶ produce no stray fields,
- ▶ feature ultrafast dynamics, and
- ▶ can generate large magnetotransport effects.

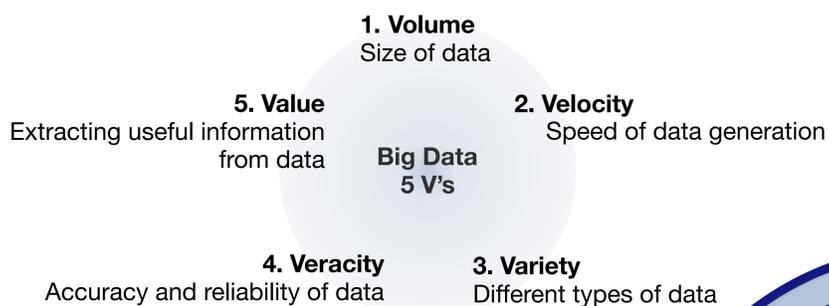
## Optimized antiferromagnetic materials can open doors for

seamless and low-maintenance energy generation [9]

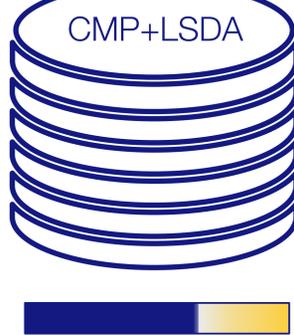
ultrafast spintronics and robust data retention [3,8]

fundamental understanding of magnetotransport [2,10-12]

Within the data-driven paradigm, **material discovery** [5] can be seen as a learning problem. That necessitates building a database of materials and the desired descriptor, for instance this database might contain crystals and their magnetic configurations.



**MAGNDATA** [4] is a collection of experimentally determined magnetic structures with portable cif-type files. This sets the path for developing a reliable scheme of computational data generation that obeys the 5 V's of Big Data.



## Cluster multipole (CMP) expansion [1, 12-13]

Vector Poisson equation :  $\nabla^2 \mathbf{A}(\mathbf{r}) = -\frac{4\pi}{c} \mathbf{j}(\mathbf{r})$

current density:  $\mathbf{j}(\mathbf{r}) = \mathbf{c} \nabla \times \mathbf{M}(\mathbf{r})$ , magnetization density:  $\mathbf{M}(\mathbf{r})$

Vector gauge potential:  $\mathbf{A}(\mathbf{r}) = \sum_{lm} b_l M_{lm} \frac{\mathbf{Y}_{lm}^l}{r^{l+1}}$ ,

vector spherical harmonics :  $\mathbf{Y}_{lm}^l = \frac{\mathbf{1} Y_m^l(\theta, \phi)}{il}$  ( $l \geq 1, -l \leq m \leq l$ ),

Coulomb gauge :  $\nabla \cdot \mathbf{A}(\mathbf{r}) = \mathbf{0}$ .

Magnetic cluster multipole (CMP):  $M_{lm} = \sqrt{\frac{4\pi}{2l+1}} \sum_{i=1}^{N_{atom}} \mathbf{m}_i \cdot \left[ \nabla \left( \mathbf{r}_i Y_{lm}^*(\theta, \phi) \right) \right]$ .

magnetic configuration  $|m\rangle = (\mathbf{m}_1, \mathbf{m}_2, \dots, \mathbf{m}_{N_{atom}})^T$

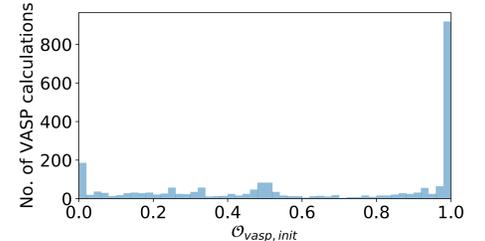
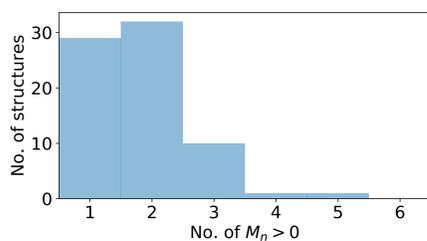
unit cell of  $N_{atom}$  magnetic sites.

Symmetrization according to irreducible representations of the crystallographic point group

$$\mathcal{Y}_{l\gamma} = \sum_m c_{lm}^\gamma Y_{lm}, \quad \gamma = 1, \dots, 2l+1, \quad l\gamma \mapsto n$$

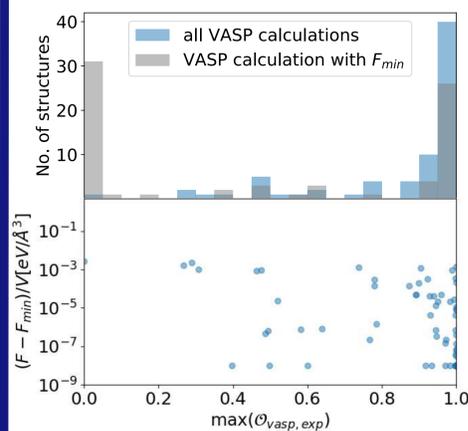
## Preliminary results

▶ For 73 (of 171) structures containing 3d transition metals on MAGNDATA with  $\mathbf{q} = \mathbf{0}$ .



## The CMP expansion is a natural expansion for experimental magnetic structures.

▶ Only about 1-3 CMP coefficients are finite among  $3N_{atom}$  magnetic configurations in the CMP basis.



## The CMP basis and specific linear combinations with same rank and irreducible representation administer an exhaustive list of candidate magnetic structures.

▶ Most converged VASP calculations have a large overlap with the initial magnetic configuration;  $\mathcal{O}_{vasp,init} \approx 1$ .

▶ Some initial magnetic configurations have almost no overlap with the converged VASP calculation;  $\mathcal{O}_{vasp,init} \approx 0$ . Presumably, they are very unstable, because they might correspond to local maxima in the LSDA free energy landscape.

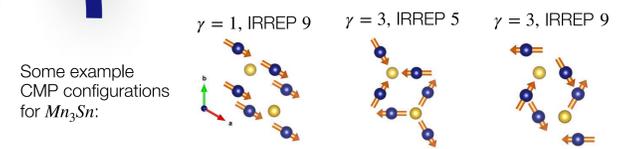
A few initial magnetic configurations converge towards a magnetic configuration that is a linear configuration of itself and another CMP with likely the same irreducible representation.

## CMP+LSDA can find the local minimum in the LSDA free energy landscape closely resembling the experimental magnetic structure.

▶ For most structures the maximum overlap among all VASP calculations w. the experimental configuration is close to 1; (blue).

▶ The VASP calculation w. lowest found free energy  $F_{min}$  does often *not* correspond to the experiment; (blue vs. gray).

▶ The structures with similar energy gaps, that is btw. the  $F_{min}$  configuration and the local minimum closest to the experimental configuration, are currently being analyzed for commonalities.



Some example CMP configurations for  $Mn_3Sn$ :

Cluster multipole basis set  $\{ |n\rangle = (\mathbf{e}_1^n, \mathbf{e}_2^n, \dots, \mathbf{e}_{N_{atom}}^n)^T \}$ ,

complete :  $\sum_{n=1}^{3N_{atom}} \frac{|n\rangle \langle n|}{N_{atom}} = \mathbf{1}$ , orthonormal :  $\langle n | n' \rangle = \delta_{nn'} N_{atom}$

symmetry-adapted CMP:  $M_n = \sum_{i=1}^{N_{atom}} \mathbf{m}_i \cdot \mathbf{e}_i^n = \langle m | n \rangle = \langle n | m \rangle$

arbitrary magnetic configuration:  $|m\rangle = \frac{1}{N_{atom}} \sum_{n=1}^{N_{atom}} M_n |n\rangle$ .

overlap of magnetic configurations:  $\mathcal{O}_{mm'} = \left( \frac{\langle m | m' \rangle}{\sqrt{\langle m | m \rangle} \sqrt{\langle m' | m' \rangle}} \right)^2$

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