In-Hadron Condensates in an Analytic Salpeter Approach

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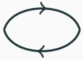
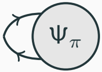
Condensates

vacuum	*in-hadron*
$\langle 0\|\overline{q}q\|0\rangle$	$\langle 0\|\overline{q}\gamma^5 q\|\pi\rangle$
	

Gell-Mann-Oakes-Renner Relation

$$M_\pi^2 f_\pi = 2m \frac{\langle 0|\overline{q}q|0\rangle}{f_\pi}$$

Gell-Mann, Oakes, Renner, *Phys. Rev.* 175 (1968)

Gell-Mann-Oakes-Renner Relation

$$M_\pi^2 f_\pi = 2m \frac{\langle 0|\overline{q}q|0\rangle}{f_\pi}$$

$$\downarrow$$

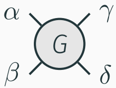
$$= 2m \quad \mathbb{C}_\pi$$

Gell-Mann, Oakes, Renner, *Phys. Rev.* **175** (1968)

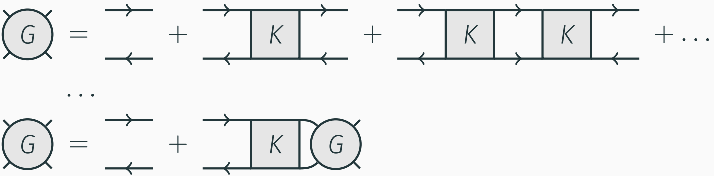
Dyson Equation

4-point Green's function

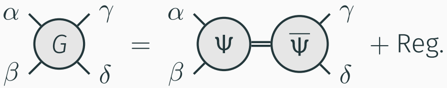
$$G^{(4)}_{\alpha\beta\gamma\delta}(x_1, x_2; y_1, y_2) = \langle 0|\, \mathrm{T}\psi_\alpha(x_1)\overline{\psi}_\beta(x_2)\overline{\psi}_\gamma(y_1)\psi_\delta(y_2)\,|0\rangle =$$



Dyson equation



Bound States $\leftrightarrow$ Poles

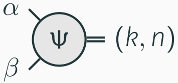


$$\lim_{P^0 \to M} G = \frac{\Psi \overline{\Psi}}{2P^0\, \epsilon} \qquad (\epsilon \to 0^+)$$

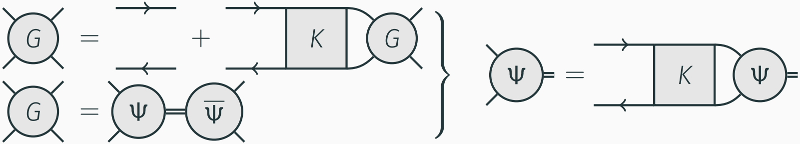
Bethe-Salpeter Equation

Mesonic Bethe-Salpeter amplitude

$$\Psi_{\alpha\beta}(x_1, x_2; \mathbf{k}, n) = \langle 0| \, \mathrm{T}\psi_\alpha(x_1)\overline{\psi}_\beta(x_2) \, |k, n\rangle$$



Bethe-Salpeter equation



Ansatz & Assumptions

$$\Phi(\mathsf{p}) = \left[\varphi_1(\mathsf{p})\frac{H(\mathsf{p})}{E(\mathsf{p})} + \varphi_2(\mathsf{p})\right]\gamma^5$$

$$\hat{K}_{\alpha\beta\gamma\delta}(\mathsf{p},\mathsf{q}) = \sum_{\Gamma,\Gamma'} V_{\Gamma\Gamma'}(\mathsf{p},\mathsf{q})\Gamma_{\alpha\gamma}\otimes\Gamma'_{\delta\beta}$$

- Instantaneous interactions: $\Psi \to \Phi$
  - Φ describes a pseudoscalar particle
- Kernel: scalar potential and Dirac structure
  - Fierz invariance
  - $\Gamma = \Gamma'$

Olsson, Veseli, Williams, *PRD* **52** (1995); $\Gamma \in \{1, \mathrm{i}\gamma^5, \gamma^\mu, \gamma^\mu\gamma^5, \sigma^{\mu\nu}\}$

Interaction Kernel

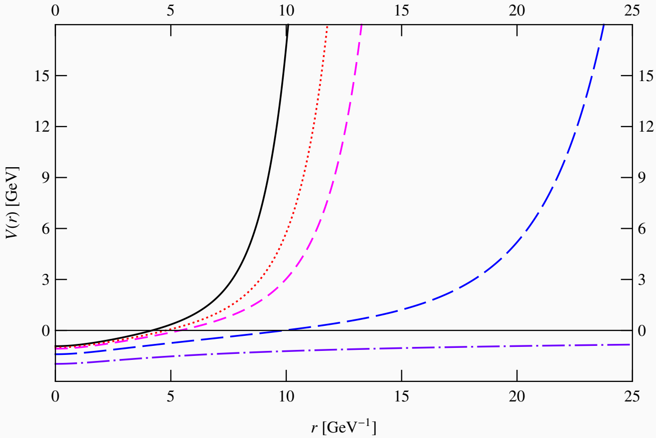


Figure 1: Spatial interaction for lightest pseudoscalar mesons

Gell-Mann-Oakes-Renner Relation

$$M_\pi^2 f_\pi = 2m\,\mathbb{C}_\pi$$

Solve Bethe-Salpeter Equation

$$M^2\varphi_2(p) = 4E^2(p)\varphi_2(p) + \frac{8}{\pi}\frac{E(p)}{p}\int\limits_{q_+} q\,\varphi_2(q)\int\limits_{r_+} \sin(pr)\sin(qr)V(r)$$

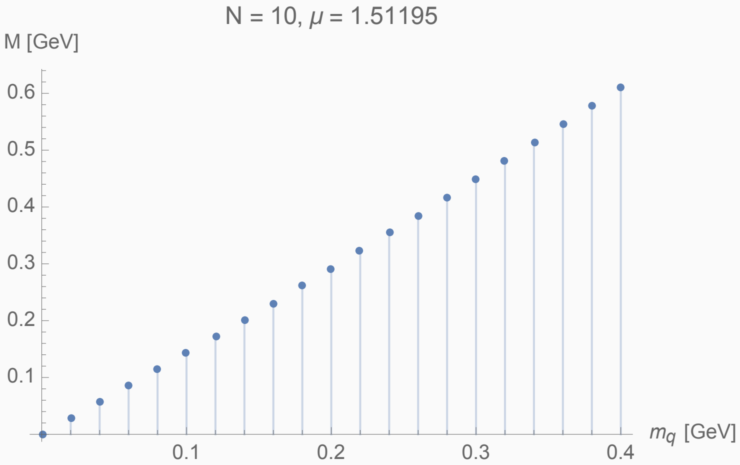
Solve Bethe-Salpeter Equation

$$M^2\varphi_2(p) = 4E^2(p)\varphi_2(p) + \frac{8}{\pi}\frac{E(p)}{p}\int_{q_+} q\,\varphi_2(q)\int_{r_+}\sin(pr)\sin(qr)V(r)$$

$$\downarrow$$

$$\mathcal{M}_{ij}v_j = M^2 v_i$$

Results – Mass



Gell-Mann-Oakes-Renner Relation

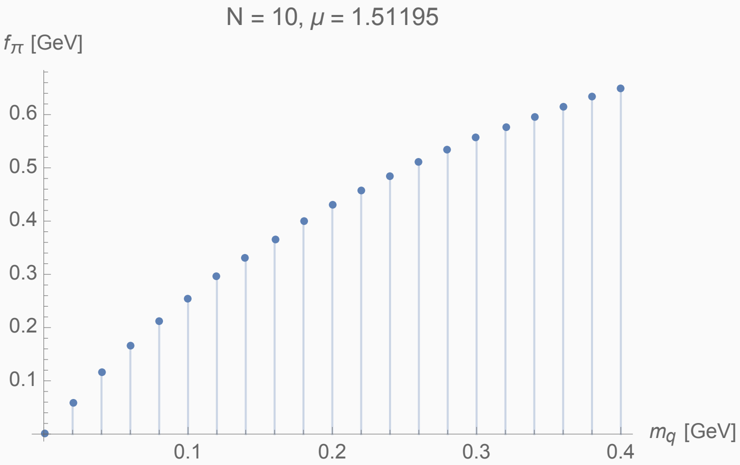
$$M_\pi^2 f_\pi = 2m\,\mathbb{C}_\pi$$

Decay Constant

$$P^\mu f_\pi = \langle 0|\overline{q}\gamma^\mu\gamma^5 q|\pi(P)\rangle$$

$$= \mathrm{Tr}\,\gamma^\mu\gamma^5 \underbrace{\langle 0|\overline{q}\otimes q|\pi(P)\rangle}_{\Psi_\pi}$$

Results – Decay Constant



Gell-Mann-Oakes-Renner Relation

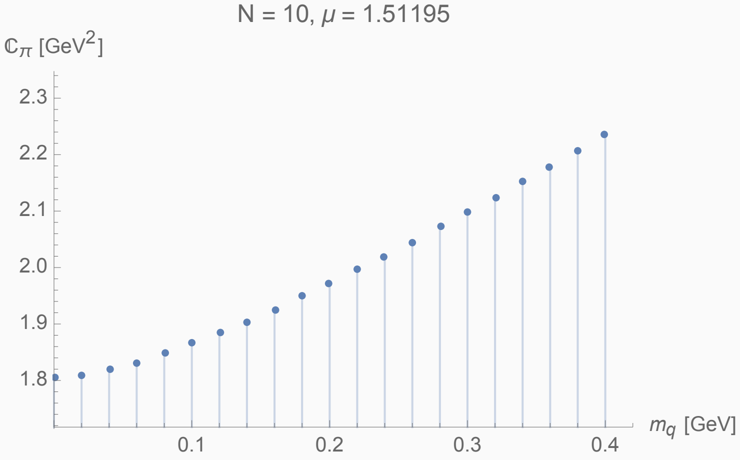
$$M_\pi^2 f_\pi = 2m\,\mathbb{C}_\pi$$

In-Hadron Condensate

$$\mathbb{C}_\pi = \langle 0|\overline{q}\mathrm{i}\gamma^5 q|\pi(P)\rangle$$

$$= \mathrm{Tr}\,\mathrm{i}\gamma^5 \underbrace{\langle 0|\overline{q} \otimes q|\pi(P)\rangle}_{\Psi_\pi}$$

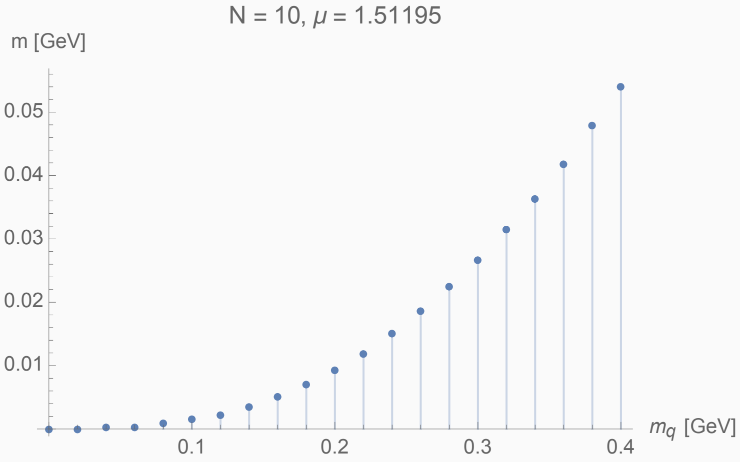
Results – In-Hadron Condensate



Gell-Mann-Oakes-Renner Relation

$$M_\pi^2 f_\pi = 2m\,\mathbb{C}_\pi$$

Results – Comparing to GMOR



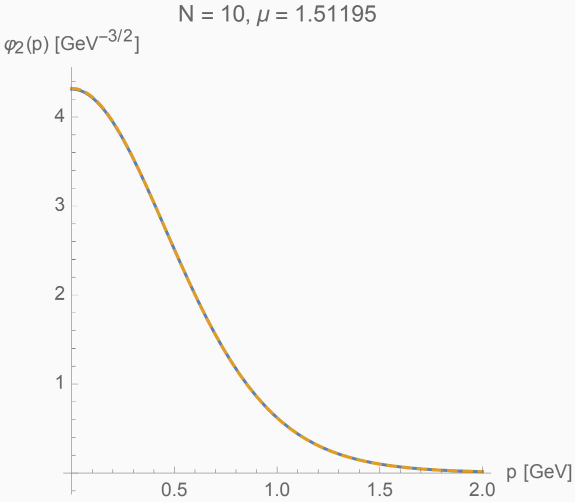
Thank you!

Summary

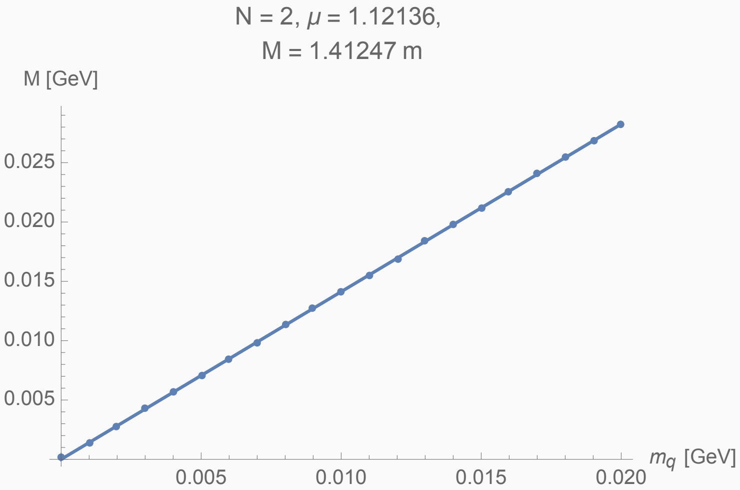
- Investigate GMOR relation
  - $\langle \overline{q}q \rangle \to \mathbb{C}$
- Use Bethe-Salpeter formalism
  - Calculate M_π, f_π, $\mathbb{C}_\pi$
- Generalized GMOR relation valid in chiral limit
- Results slightly larger as expected
  - Broader potential $\to$ lower eigenvalues

Backup

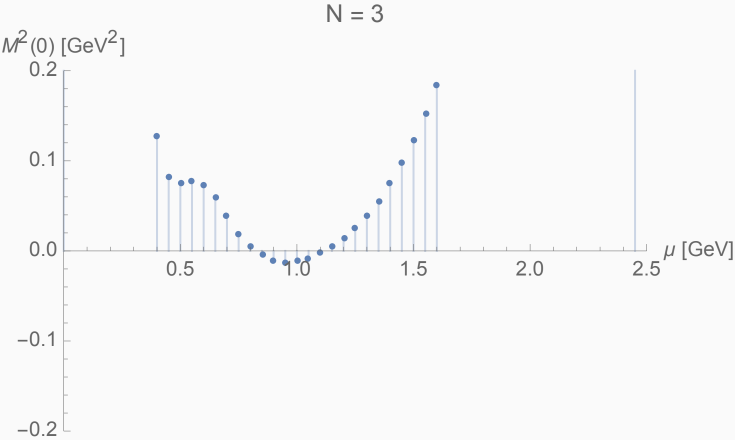
Results – Wave Function



Results – M_π near Origin



Choice of μ



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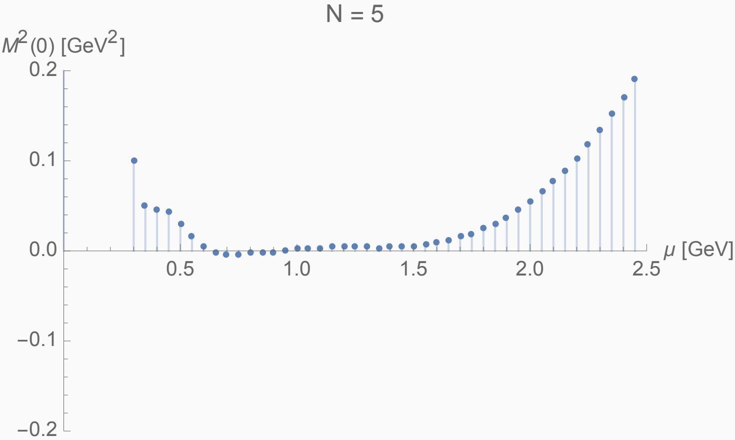
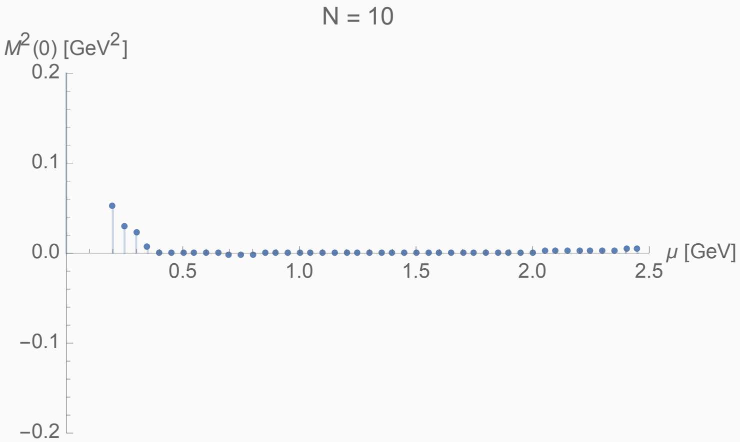
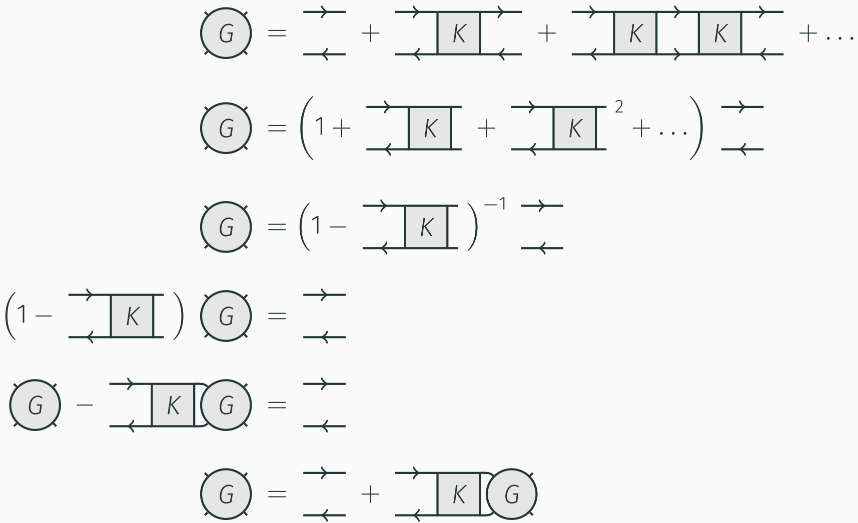


Choice of μ



Dyson Equation



Normalization

$$P^{\mu} = \overline{\Psi} \left(\frac{\partial}{\partial P_{\mu}} (S_1 \otimes S_2) - S_1 \, S_2 \, \left(\frac{\partial}{\partial P_{\mu}} K \right) S_1 \, S_2 \right) \Psi$$

References i

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