



HEPHY

Institut für Hochenergiephysik



In-Hadron Condensates in an Analytic Salpeter Approach

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Condensates

vacuum

$$\langle 0 | \bar{q} q | 0 \rangle$$



in-hadron

$$\langle 0 | \bar{q} \gamma^5 q | \pi \rangle$$



Gell-Mann-Oakes-Renner Relation


$$M_\pi^2 f_\pi = 2m \frac{\langle 0 | \bar{q}q | 0 \rangle}{f_\pi}$$

Gell-Mann-Oakes-Renner Relation

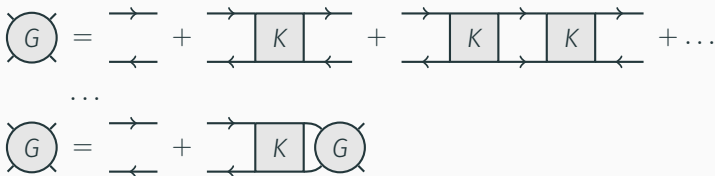
$$\begin{aligned} M_\pi^2 f_\pi &= 2m \frac{\langle 0 | \bar{q}q | 0 \rangle}{f_\pi} \\ &\quad \downarrow \\ &= 2m \quad \mathbb{C}_\pi \end{aligned}$$

Dyson Equation

4-point Green's function

$$G_{\alpha\beta\gamma\delta}^{(4)}(x_1, x_2; y_1, y_2) = \langle 0 | T \psi_{\alpha}(x_1) \bar{\psi}_{\beta}(x_2) \bar{\psi}_{\gamma}(y_1) \psi_{\delta}(y_2) | 0 \rangle =$$


Dyson equation



The diagram illustrates the Dyson equation for a Green's function G . It shows two equations:

1. $G = \text{free propagator} + \text{one K vertex} + \text{two K vertices} + \dots$

2. $G = \text{free propagator} + \text{one K vertex} + \text{one K vertex} + G$

Bound States \leftrightarrow Poles

The diagram shows an equality between two Feynman diagrams. On the left, a single vertex labeled G is represented by a circle with four external lines labeled α , β , γ , and δ . On the right, the vertex G is decomposed into a pole term and a regular part. The pole term consists of two vertices, Ψ and $\bar{\Psi}$, connected by a double horizontal line. The Ψ vertex has external lines α and β , and the $\bar{\Psi}$ vertex has external lines γ and δ . The regular part is denoted as $+ \text{Reg.}$.

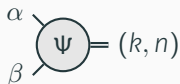
$$\alpha \begin{array}{c} \diagup \\ \diagdown \end{array} \begin{array}{c} \diagdown \\ \diagup \end{array} G \begin{array}{c} \diagup \\ \diagdown \end{array} \begin{array}{c} \diagdown \\ \diagup \end{array} \gamma \beta \delta = \alpha \begin{array}{c} \diagup \\ \diagdown \end{array} \begin{array}{c} \diagdown \\ \diagup \end{array} \Psi \begin{array}{c} \diagdown \\ \diagup \end{array} \begin{array}{c} \diagdown \\ \diagup \end{array} \bar{\Psi} \begin{array}{c} \diagup \\ \diagdown \end{array} \begin{array}{c} \diagdown \\ \diagup \end{array} \gamma \beta \delta + \text{Reg.}$$

$$\lim_{P^0 \rightarrow M} G = \frac{\Psi \bar{\Psi}}{2P^0 \epsilon} \quad (\epsilon \rightarrow 0^+)$$

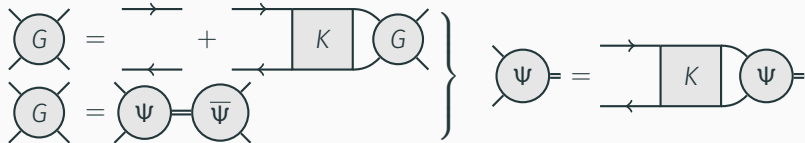
Bethe-Salpeter Equation

Mesonic Bethe-Salpeter amplitude

$$\Psi_{\alpha\beta}(x_1, x_2; \mathbf{k}, n) = \langle 0 | T \psi_\alpha(x_1) \bar{\psi}_\beta(x_2) | \mathbf{k}, n \rangle$$



Bethe-Salpeter equation



$$\Phi(\mathbf{p}) = \left[\varphi_1(\mathbf{p}) \frac{H(\mathbf{p})}{E(\mathbf{p})} + \varphi_2(\mathbf{p}) \right] \gamma^5$$

$$\hat{K}_{\alpha\beta\gamma\delta}(\mathbf{p}, \mathbf{q}) = \sum_{\Gamma, \Gamma'} V_{\Gamma\Gamma'}(\mathbf{p}, \mathbf{q}) \Gamma_{\alpha\gamma} \otimes \Gamma'_{\delta\beta}$$

- Instantaneous interactions: $\Psi \rightarrow \Phi$
 - Φ describes a pseudoscalar particle
- Kernel: scalar potential and Dirac structure
 - Fierz invariance
 - $\Gamma = \Gamma'$

Interaction Kernel

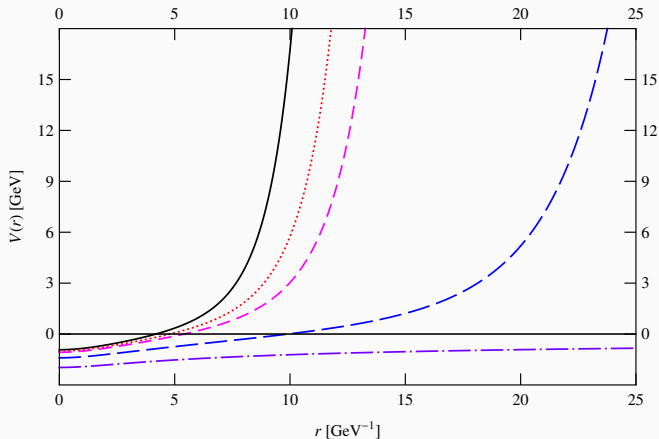


Figure 1: Spatial interaction for lightest pseudoscalar mesons

Gell-Mann-Oakes-Renner Relation

$$M_\pi^2 f_\pi = 2m C_\pi$$

Solve Bethe-Salpeter Equation

$$M^2 \varphi_2(p) = 4E^2(p) \varphi_2(p) + \frac{8}{\pi} \frac{E(p)}{p} \int_{q_+} q \varphi_2(q) \int_{r_+} \sin(pr) \sin(qr) V(r)$$

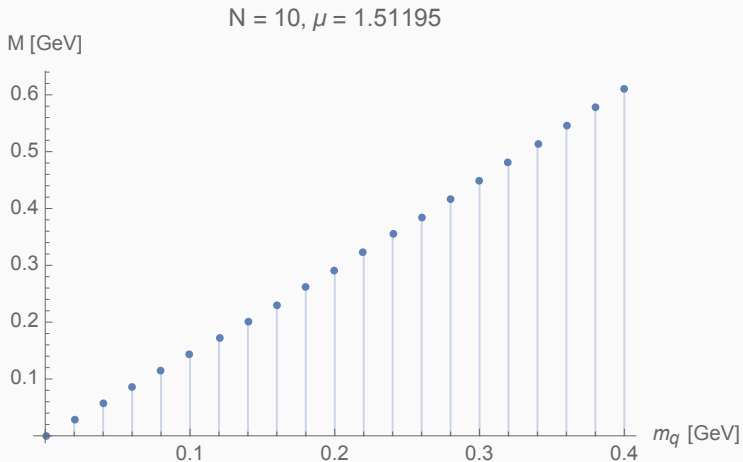
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↓

$$\mathcal{M}_{ij} v_j = M^2 v_i$$

Results – Mass

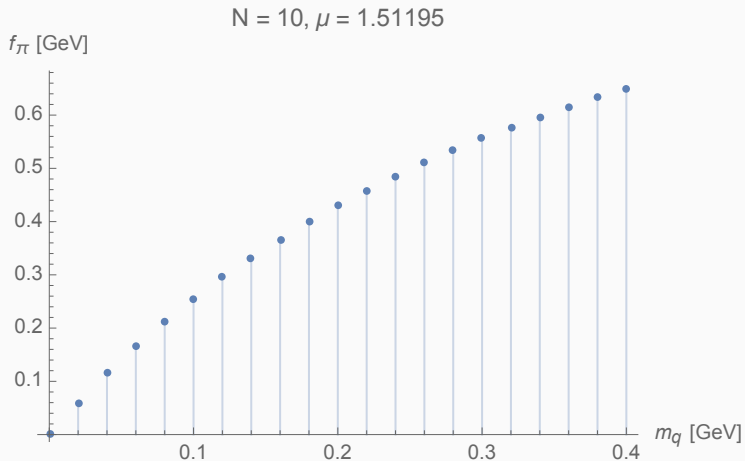


Gell-Mann-Oakes-Renner Relation

$$M_\pi^2 f_\pi = 2m \mathbb{C}_\pi$$

$$\begin{aligned} P^\mu f_\pi &= \langle 0 | \bar{q} \gamma^\mu \gamma^5 q | \pi(P) \rangle \\ &= \text{Tr} \gamma^\mu \gamma^5 \underbrace{\langle 0 | \bar{q} \otimes q | \pi(P) \rangle}_{\Psi_\pi} \end{aligned}$$

Results – Decay Constant

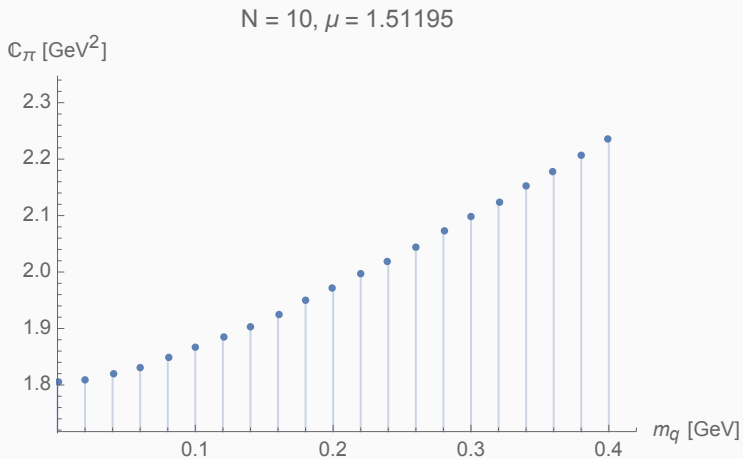


Gell-Mann-Oakes-Renner Relation

$$M_\pi^2 f_\pi = 2m \mathbb{C}_\pi$$

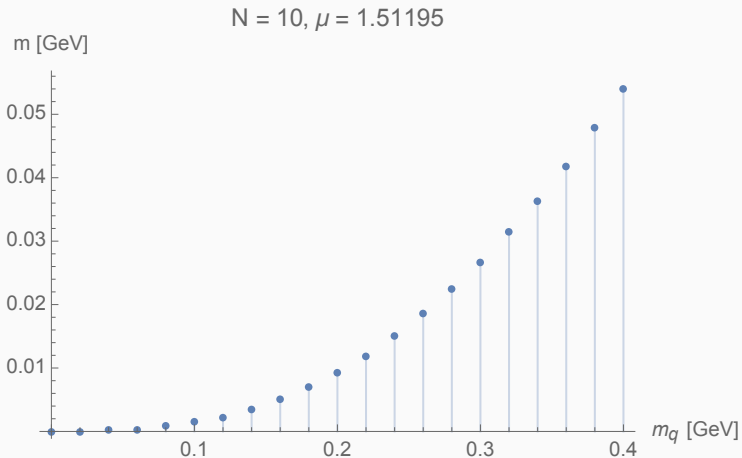
$$\begin{aligned} C_\pi &= \langle 0 | \bar{q} i \gamma^5 q | \pi(P) \rangle \\ &= \text{Tr} i \gamma^5 \underbrace{\langle 0 | \bar{q} \otimes q | \pi(P) \rangle}_{\Psi_\pi} \end{aligned}$$

Results – In-Hadron Condensate



$$M_\pi^2 f_\pi = 2m C_\pi$$

Results – Comparing to GMOR



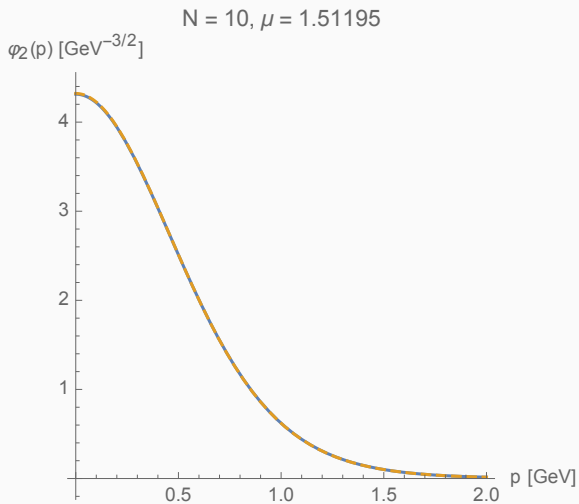
Thank you!

Summary

- Investigate GMOR relation
 - $\langle \bar{q}q \rangle \rightarrow \mathbb{C}$
- Use Bethe-Salpeter formalism
 - Calculate $M_\pi, f_\pi, \mathbb{C}_\pi$
- Generalized GMOR relation valid in chiral limit
- Results slightly larger as expected
 - Broader potential \rightarrow lower eigenvalues

Backup

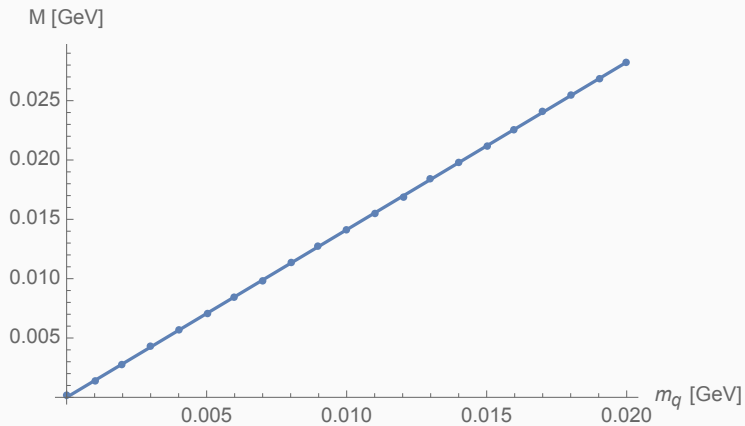
Results – Wave Function



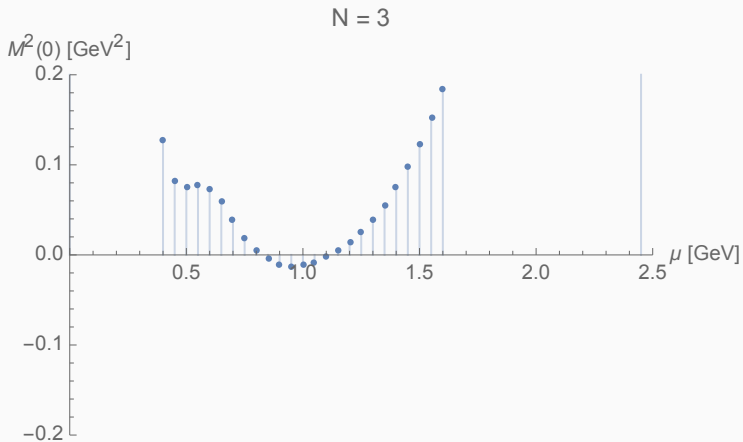
Results – M_π near Origin

$N = 2, \mu = 1.12136,$

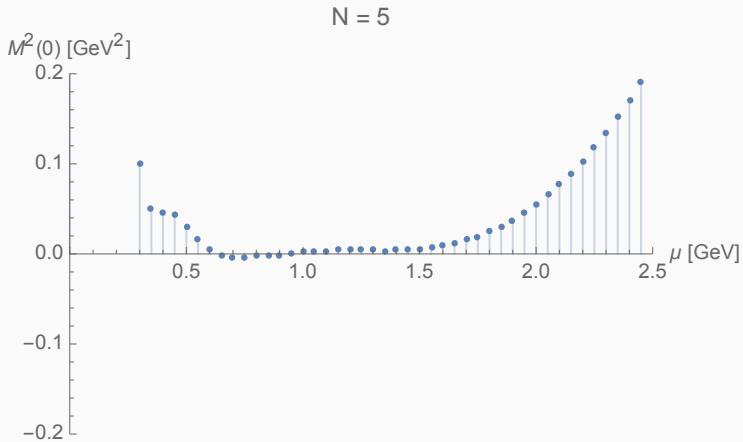
$M = 1.41247 m$



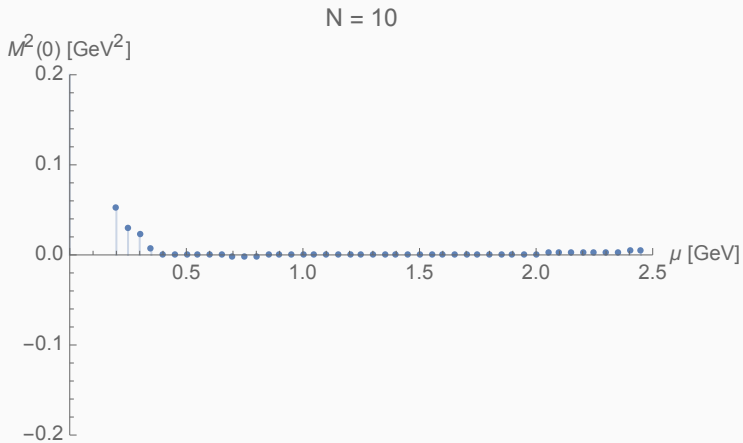
Choice of μ



Choice of μ



Choice of μ



Dyson Equation

$$\textcircled{G} = \begin{array}{c} \rightarrow \\ \leftarrow \end{array} + \begin{array}{c} \rightarrow \\ \leftarrow \end{array} \boxed{K} \begin{array}{c} \rightarrow \\ \leftarrow \end{array} + \begin{array}{c} \rightarrow \\ \leftarrow \end{array} \boxed{K} \boxed{K} \begin{array}{c} \rightarrow \\ \leftarrow \end{array} + \dots$$

$$\textcircled{G} = \left(1 + \begin{array}{c} \rightarrow \\ \leftarrow \end{array} \boxed{K} + \begin{array}{c} \rightarrow \\ \leftarrow \end{array} \boxed{K}^2 + \dots \right) \begin{array}{c} \rightarrow \\ \leftarrow \end{array}$$

$$\textcircled{G} = \left(1 - \begin{array}{c} \rightarrow \\ \leftarrow \end{array} \boxed{K} \right)^{-1} \begin{array}{c} \rightarrow \\ \leftarrow \end{array}$$

$$\left(1 - \begin{array}{c} \rightarrow \\ \leftarrow \end{array} \boxed{K} \right) \textcircled{G} = \begin{array}{c} \rightarrow \\ \leftarrow \end{array}$$

$$\textcircled{G} - \begin{array}{c} \rightarrow \\ \leftarrow \end{array} \boxed{K} \textcircled{G} = \begin{array}{c} \rightarrow \\ \leftarrow \end{array}$$

$$\textcircled{G} = \begin{array}{c} \rightarrow \\ \leftarrow \end{array} + \begin{array}{c} \rightarrow \\ \leftarrow \end{array} \boxed{K} \textcircled{G}$$

Normalization

$$P^\mu = \bar{\Psi} \left(\frac{\partial}{\partial P_\mu} (S_1 \otimes S_2) - S_1 S_2 \left(\frac{\partial}{\partial P_\mu} K \right) S_1 S_2 \right) \Psi$$

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