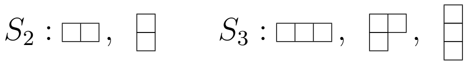
Representations of $SU(N)$ – Young Diagrams

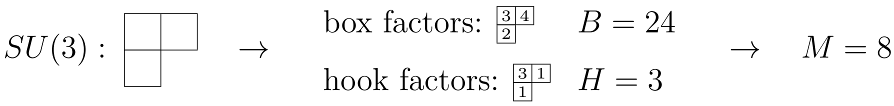
Young diagrams. The permutation group S_n can be visualized using n boxes that are put together in all possible ways, following the rules that each row has to have the same or less boxes as the row above and each column has to have the same or less boxes as the column to its left. For example:



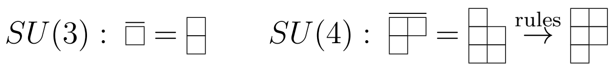
A horizontal (vertical) connection stands for (anti-)symmetrization.

Representations of $SU(N)$. Every Young diagram corresponds to a representation of $SU(N)$. The most important ones are:

The multiplicity[1] can be calculated as $M = B/H$. Here, $B = \Pi\, b_i$ is the box factor. The box to the top-left gets assigned the value $b_1 = N$. The other boxed get filled according to the rules that a step to the right increases the number and a step down decreases the number. $H = \Pi\, h_i$ is the hook factor. Each box gets assigned a value h, which is the total number of all neighboring boxes to the right and below plus one for itself. For example:

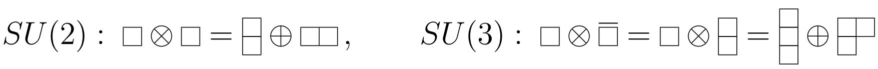


Complex conjugate representation. The conjugate representation $\overline{R}$ of a Young diagram R can be described by changing each column from x boxes to $N - x$ boxes and then flipping the image with respect to its vertical axis. If the rules are violated, move the boxes up or to the left such that the rules are fulfilled again. For example:



In order to know whether □□ is the $[6]$ or $[\bar{6}]$ representation of $SU(3)$, the rule of thumb is that if the height of the diagram is less than $N/2$, then is is the unbarred representation. If the height is exactly $N/2$, like □ for $SU(2)$, then $R = \overline{R}$ and it is a real representation.

Tensor product. In order to multiply two Young diagrams, we connect the second one in every allowed way to the first one and add these terms. If necessary, we exchange a conjugated diagram according to the rules in the previous section. For example:



[1] The number of states in each irreducible representation.